A STACKELBERG GAME-THEORETIC COOPERATIVE ADVERTISING MODEL: THE EFFECT OF PLAYERS' STRATEGIES IN A THREE-MEMBER CHANNEL

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Abstract: This work considers cooperative advertising in a manufacturer-distributor-retailer supply chain. The manufacturer is the Stackelberg leader. The distributor is the first follower while the retailer is the second follower. The game structure is on a setting in which only the retailer advertises the product while the distributor and the manufacturer indirectly participate in retail advertising by providing subsidies. The work considers a four-game scenario: a situation where neither the distributor nor the manufacturer participates in retail advertising; a situation where only the distributor participates in retail advertising; a situation where only the manufacturer participates in retail advertising; and a situation where both the distributor and the manufacturer participate in retail advertising. In each of these cases the work obtains the optimal advertising strategy, the optimal subsidy (participation) strategies (rates), and the players' payoffs. The work considers the effect of the players' strategies on the payoffs. While the manufacturer and/or distributor's payoff reduce(s) with increasing participation rate(s), the other players' payoffs increase and become very large. Similarly, as the retail advertising effort increases in the absence of subsidy, the other players' payoffs become very large, and only tend to reduce with their subsidised retail advertising effort. For any given structure a player should stick to his optimal strategy if he must not be short changed by the other players.

Keywords: Cooperative advertising, Stackelberg game, subsidy rate, supply channel

Introduction

Cooperative advertising is usually employed by the manufacturer as a motivational strategy towards the retailer in supply chains. It is an arrangement in which the manufacturer contributes a certain percentage of the retail advertising expenditure incurred in the course of advertising. Classical cooperative advertising models usually assume that cooperative advertising supply channel consists of only the manufacturer(s) and the retailer(s). But in real life setting there is usually a link in the form of a middleman which is a third party between these two parties. This is the distributor. Thus the manufacturer sells to the distributor who in turn sells to the retailer. The retailer sells the product to the end-users by engaging in retail advertising. This work develops for the first time a cooperative advertising model in which the distributor is incorporated to assist the manufacturer in subsidising the retail advertising effort on a static setting. Thus both the distributor and the manufacturer participate in retail advertising by subsidising retail advertising effort. The manufacturer is the Stackelberg leader. The distributor is the first follower, while the retailer is the second (last) follower.

Considering static models can aid the development of analytic solutions and understanding of the role of basic parameters such as the players' margins, retail advertising effort and manufacturer and distributor's participation rates. Berger (1972) is known to be the first static cooperative advertising game model. He proposed a model in which cooperative advertising is a wholesale price discount from the manufacturer to his retailer. Dant and Berger (1996) extended Berger (1972) by considering a franchising system where there is demand uncertainty. Noting that there is a substantial difference between local (retail) and national (manufacturer's) advertising expenditure Huang and Li (2001), Huang *et al.* (2002) and Li *et al.* (2002) considered advertising strategies for different types of relationships between the retailer and the manufacturer. Yue *et al.* (2006) in an extension of Huang and Li (2001) considered cooperative advertising coordination when the manufacturer offers price discount in a manufacturer-retailer channel. In another extension of Huang and Li (2001), Xie and Neyret (2009) considered optimal pricing and cooperative advertising decisions using four classical forms of relationship between a manufacturer and a retailer. Wei and Xie (2009) established and compared a leader-follower non-cooperative game model with a centralised cooperative game model; and considered channel coordination by determining optimal cooperative advertising decisions and equilibrium pricing strategies in a supply chain involving two players. He *et al.* (2014) considered the optimal cooperative decisions in a two-period fashion and textile manufacturer-retailer supply chain in which the individual channel members make decisions based on a cooperative program and a vertically integrated manufacturer-retailer relationship. They introduced a two-way subsidy contract for the coordination of the supply chain. Ezimadu (2017) considered a cooperative advertising model in which the manufacturer is the Stackelberg leader, while the competing retailers play a Nash game with each other. The competing retailers engage in local advertising, while the manufacturer engage in national advertising and also subsidises the retailers' advertising efforts. The above models are based on the traditional manufacturer(s)-retailer(s) channel.

In the traditional cooperative advertising literature the retailer engages in advertising which is subsidised by the manufacturer. This work extends the classical cooperative advertising model from a manufacturer-retailer to a manufacturer-distributor-retailer static model in a monopolistic setting on which only the retailer is involved in advertising while both the manufacturer and the distributor subsidise retail advertising. It is an extension of Ezimadu (2019) in which the distributor is indirectly involved in retail advertising through subsidy just like the manufacturer. We will consider four channel structures which includes:

(*i*) a situation where subsidy is not provided for retail advertising;
(*ii*) a situation where only the distributor provides subsidy for retail advertising;
(*iii*) a situation where only the manufacturer provides subsidy for retail advertising;
(*iv*) a situation where both the distributor and the manufacturer provide subsidy for retail advertising.

For each of these we will obtain (as the case may be) the optimal strategies: the optimal advertising effort of the retailer; and the optimal subsidy rate(s) of the manufacturer and/or the distributor. Based on this model setting – where only the retailer is involved in advertising while the manufacturer and the distributor subsidize retail advertising – we will consider the effect of the subsidies on each player's payoff. We will also consider the effect of the advertising effort on each player's payoff for each channel structure.

Model Development

In this work we will consider a single-manufacturer-distributor-retailer channel. The distributor sells only the manufacturer's brand (among substitutes in a product class) to the retailer who then sells to the consumer(s). The retailer's decision variable is his advertising expenditure α_R, while the manufacturer and distributor's decision variables are their participation rates ψ_M and ψ_D respectively.

The impact of retail advertising on demand is given by

$$D(\alpha) = \beta\sqrt{\alpha}\ , \qquad (1)$$

Where: $\beta \in [0,1]$ reflects the effect of advertising on sale. Clearly (1) is an increasing concave function. This property is consistent with mostly observed advertising saturation effect. That is every additional spending on advertising continuously results in diminishing returns (Simon and Arndt, 1980, Xie and Wei, 2009, He *et al*.., 2009, Chutani and Sethi 2012).

Now, let M_R, M_D and M_M be the price margins of the retailer, the distributor and the manufacturer respectively. Since only the retailer is involved in advertising, and receives subsidy from both the manufacturer and the distributor, we have that his profit is given by

$$R_{Pay} = M_R\beta\sqrt{\alpha} - \alpha + \alpha\psi_D + \alpha\psi_M, \qquad (2)$$

Both the distributor and the manufacturer do not directly engage in advertising, but indirectly participate through provision of subsidy in support of retail advertising. Thus their profits are given by

$$D_{Pay} = M_D\beta\sqrt{\alpha} - \alpha\psi_D\ , \qquad (3)$$

and

$$M_{Pay} = M_M\beta\sqrt{\alpha} - \alpha\psi_M, \qquad (4)$$

respectively.

The leader-followers relationship model

We will model the decisions in this section as a sequential non-cooperative game with the manufacturer as the Stackelberg game leader, the distributor as the first follower and the retailer as the last follower. First, the manufacturer decides and informs the other players of his margin M_M (from the distributor) and participation rate ψ_M to the retailer. The distributor follows by making his margin M_D (from the retailer) and participation rate ψ_D (to the retailer) known. The retailer in turn then decides on his advertising effort α and margin M_R. Thus given the distributor and manufacturer's decisions the retailer aims to

$$\begin{aligned} &\max\ R_{Pay} = M_R\beta\sqrt{\alpha} - \alpha + \alpha\psi_D + \alpha\psi_M \\ &\text{s.t.}\quad \alpha > 0. \end{aligned} \qquad (5)$$

The optimal value of ψ_D is determined by maximizing the distributor's profit. Thus his problem is given by

$$\begin{aligned} &\max\ D_{Pay} = M_D\beta\sqrt{\alpha} - \alpha\psi_D \\ &\text{s.t.}\quad 0 \le \psi_D \le 1. \end{aligned} \qquad (6)$$

Similarly, to obtain the optimal participation ψ_M the manufacturer maximizes his profit. Thus his objective is to

$$\begin{aligned} &\max\ M_{Pay} = M_M\beta\sqrt{\alpha} - \alpha\psi_M \\ &\text{s.t.}\quad 0 \le \psi_M \le 1. \end{aligned} \qquad (7)$$

The Players' Strategies

Proposition 4.1 The retailer's strategy is given by

$$\alpha = \frac{\beta^2 M_R^2}{4(1-\psi_D-\psi_M)^2}\ , \qquad (8)$$

while the distributor and manufacturer's strategies are;

$$\psi_D = \begin{cases} \dfrac{(2M_D - M_R)(1-\psi_M)}{2M_D + M_R} \\ 0 \quad \text{otherwise} \end{cases} = \begin{cases} \dfrac{2M_D - M_R}{2(M_D + M_M)}, & 2M_D > M_R,\ \psi_M \ne 0, \\ 0 \quad \text{otherwise}\ , \end{cases} \qquad (9)$$

and

$$\psi_M = \begin{cases} \dfrac{(2M_M - M_R)(1-\psi_D)}{2M_M + M_R} \\ 0 \quad \text{otherwise} \end{cases} = \begin{cases} \dfrac{2M_M - M_R}{2(M_M + M_D)}, & 2M_M > M_R,\ \psi_D \ne 0, \\ 0 \quad \text{otherwise}\ , \end{cases} \qquad (10)$$

respectively.

Proof: Maximizing (5) with respect to α we have

$$M_R\beta.\frac{1}{2\sqrt{\alpha}} - 1 + \psi_D + \psi_M = 0$$

which gives (8).

Using (8) in (6) we have

$$\begin{aligned} &\max\ D_{Pay} = \frac{M_R M_D \beta^2}{2(1-\psi_D-\psi_M)} - \frac{M_R^2\beta^2}{4(1-\psi_D-\psi_M)^2}\psi_D \\ &\text{s.t}\quad 0 \le \psi_D \le 1\ . \end{aligned} \qquad (11)$$

Maximizing (11) with respect to ψ_D we have

$$M_D = \frac{M_R}{2}\left[\frac{1+\psi_D-\psi_M}{1-\psi_D-\psi_M}\right],$$

$$\Rightarrow\quad \psi_D = \begin{cases} \dfrac{(2M_D - M_R)(1-\psi_M)}{2M_D + M_R} \\ 0 \quad \text{otherwise} \end{cases}. \qquad (12)$$

Similarly using (8) in (7) we have

$$\begin{aligned} &\max\ M_{Pay} = \frac{M_R M_M \beta^2}{2(1-\psi_D-\psi_M)} - \frac{M_R^2\beta^2}{4(1-\psi_D-\psi_M)^2}\psi_M \\ &\text{s.t}\quad 0 \le \psi_M \le 1\ . \end{aligned} \qquad (13)$$

Maximizing (13) with respect to ψ_M we have

$$M_M = \frac{M_R}{2}\left[\frac{1-\psi_D+\psi_M}{1-\psi_D-\psi_M}\right],$$

$$\Rightarrow \quad \psi_M = \begin{cases} \dfrac{(2M_M - M_R)(1-\psi_D)}{2M_M + M_R} \\ 0 \quad \text{otherwise} \end{cases}. \tag{14}$$

Thus from (12) and (14) we have

$$\psi_D = \frac{2M_D - M_R}{2M_D + M_R}(1-\psi_M) \tag{15}$$

and

$$\psi_M = \frac{2M_M - M_R}{2M_M + M_R}(1-\psi_D). \tag{16}$$

Using (16) in (15) we have

$$\psi_D = \frac{2M_D - M_R}{2M_D + M_R} - \frac{2M_D - M_R}{2M_D + M_R}\left[\frac{2M_M - M_R}{2M_M + M_R}(1-\psi_D)\right]$$

$$\Rightarrow \psi_D = \frac{2M_D - M_R}{2(M_D + M_M)}, \quad 2M_D > M_R. \tag{17}$$

Similarly using (17) in (16) we have

$$\psi_M = \frac{2M_M - M_R}{2M_M + M_R}\left(1 - \frac{2M_D - M_R}{2(M_D + M_M)}\right)$$

$$= \frac{2M_M - M_R}{2(M_D + M_M)}, \quad 2M_M > M_R. \quad \blacksquare \tag{18}$$

From (8) we observe that the retail advertising effort depends on the retail margin; the advertising effectiveness parameter and the participation rates from the distributor and the manufacturer.

It is obvious that a handy tool in the retailer's hand is his margin. As the margin increases, the advertising effort also increases. This is understandable because by the law of price and demand increase in price margin will lead to reduced demand which will eventually affect his payoff. To cushion the effect of this price increase, he would increase the advertising level.

Further, as the participation (subsidy) rates increase, the advertising effort also increases. Thus, large participation rates imply large advertising effort, and low participations imply low advertising effort. Further, we observe that it would not be ideal for the distributor to totally subsidise retail advertising. This also applies to the manufacturer. Further, both should not together totally subsidise retail advertising. From (9) and (10) we observe that total subsidy from the manufacturer will make the distributor to scheme away from providing subsidy, and vice versa. The implication from (8) is that total subsidy by any of the players would make the retail advertising effort unbounded, whereas the retailer cannot be willing to spend as much as that on advertising.

Equations (9) and (10) show that the distributor's participation depends on that of the manufacturer, and vice versa. Further, the distributor will participate in retail advertising if the distributor's margin is twice larger than the retailer's margin. This is also the case with manufacturer's participation. In addition the distributor does not totally subsidise retail advertising. This is also the case with the manufacturer.

Let the subscripts $\psi_D = 0$, $\psi_M = 0$, $\psi_D > 0$, $\psi_M > 0$ represent "the distributor does not provide subsidy"; "the manufacturer does not provide subsidy"; "the distributor provides subsidy"; and "the manufacturer provides subsidy" respectively.

Players' Equilibrium Strategies and the Players' Payoffs

***Proposition 5.1 (Equilibrium Characterising Non-Provision* of Subsidy)**

Suppose that neither the manufacturer nor the distributor is involved in retail advertising, then the retail advertising effort is given by

$$\alpha_{\psi_D,\psi_M=0} = \frac{\beta^2 M_R^2}{4}, \tag{19}$$

and the resulting payoffs are

$$R_{Pay(\psi_D,\psi_M=0)} = \frac{\beta^2 M_R^2}{4}, \tag{20}$$

$$D_{Pay(\psi_D,\psi_M=0)} = \frac{\beta^2 M_R M_D}{2}, \tag{21}$$

$$M_{Pay(\psi_D,\psi_M=0)} = \frac{\beta^2 M_R M_M}{2}. \tag{22}$$

Proof: Since there is no participation we have that $\psi_D = \psi_M = 0$. As such (8) becomes (19). Using (19) in (2), (3) and (4) with $\psi_D = \psi_M = 0$, we have (20), (21) and (22), respectively. ■

From (19) we observe that without subsidy only the retailer's margin determines the advertising effort. This is a reflection of the lone-effort of the retailer in bearing the advertising burden of the entire supply chain.

Considering (20), (21) and (22) we observe a reflection of this non-provision of subsidy resulting from the lone-burden bearing of the retailer. Particularly we observe that only the retailer's margin is common to all the players' payoffs unlike what we will see in the next session where at least one player is involved in advertising. Thus in the absence of subsidy, the retailer is very influential on the payoffs

Proposition 5.2 (Equilibrium Characterising the Provision of Subsidy by only the Distributor)

Suppose that only the distributor participates in retail advertising then the subsidy rate and the retail advertising effort are given by

$$\psi_D = \frac{2M_D - M_R}{2M_D + M_R} \tag{23}$$

and

$$\alpha_{(\psi_D>0,\psi_M=0)} = \left[\frac{\beta(2M_D + M_R)}{4}\right]^2 \tag{24}$$

respectively, and the resulting payoffs are

$$R_{Pay(\psi_D>0,\psi_M=0)} = \frac{\beta^2 M_R(2M_D + M_R)}{8}, \tag{25}$$

$$D_{Pay(\psi_D>0,\psi_M=0)} = \left[\frac{\beta(2M_D + M_R)}{4}\right]^2, \tag{26}$$

$$M_{Pay(\psi_D>0,\psi_M=0)} = \frac{\beta^2 M_M(2M_D + M_R)}{4}. \tag{27}$$

Proof: Since , $\psi_M = 0$ (9) becomes

$$\psi_D = \frac{2M_D - M_R}{2M_D + M_R}.$$

Using $\psi_M = 0$ and (23) in (8) we have

$$\alpha_{(\psi_D>0,\psi_M=0)} = \frac{\beta^2 M_R^2}{4\left(1 - \dfrac{2M_D - M_R}{2M_D + M_R}\right)^2}.$$

This leads to (24).

Using $\psi_M = 0$, (23) and (24) in (5), (6) and (7) we have

$$R_{Pay(\psi_D>0,\psi_M=0)} = M_R\beta\frac{\beta(2M_D+M_R)}{4} - \left[\frac{\beta(2M_D+M_R)}{4}\right]^2\left[1-\frac{2M_D-M_R}{2M_D+M_R}\right],$$

$$D_{Pay(\psi_D>0,\psi_M=0)} = M_D\beta\frac{\beta(2M_D+M_R)}{4} - \left[\frac{\beta(2M_D+M_R)}{4}\right]^2\left[\frac{2M_D-M_R}{2M_D+M_R}\right]$$

and

$$M_{Pay(\psi_D>0,\psi_M=0)} = M_M\beta\frac{\beta(2M_D+M_R)}{4}.$$

respectively, leading to (25), (26) and (27) respectively

Proposition 5.3 (Equilibrium Characterising the Provision of Subsidy by only the Manufacturer)

Suppose that only the Manufacturer participates in retail advertising then the subsidy rate and the retail advertising effort are given by

$$\psi_M = \frac{2M_M-M_R}{2M_M+M_R} \tag{28}$$

and

$$\alpha_{(\psi_D=0,\psi_M>0)} = \left[\frac{\beta(2M_M+M_R)}{4}\right]^2 \tag{29}$$

respectively, and the resulting payoffs are

$$R_{Pay(\psi_D=0,\psi_M>0)} = \frac{\beta^2M_R(2M_M+M_R)}{8}, \tag{30}$$

$$D_{Pay(\psi_D=0,\psi_M>0)} = \frac{\beta^2M_D(2M_M+M_R)}{4}, \tag{31}$$

$$M_{Pay(\psi_D=0,\psi_M>0)} = \left[\frac{\beta(2M_M+M_R)}{4}\right]^2. \tag{32}$$

Proof: Since $\psi_D = 0$ (10) becomes

$$\psi_M = \frac{2M_M-M_R}{2M_M+M_R}.$$

Using $\psi_D = 0$ and (28) in (8) we have

$$\alpha_{(\psi_D=0,\psi_M>0)} = \frac{\beta^2M_R^2}{4\left(1-\frac{2M_M-M_R}{2M_M+M_R}\right)^2},$$

which gives (29).

Using $\psi_D = 0$, (28) and (29) in (5) we have

$$R_{Pay(\psi_D=0,\psi_M>0)} = \beta M_R\frac{\beta(2M_M+M_R)}{4} - \left[\frac{\beta(2M_M+M_R)}{4}\right]^2\left[1-\frac{2M_M-M_R}{2M_M+M_R}\right]$$

which gives (30).

Using $\psi_D = 0$ and (29) in (6) we have

$$D_{Pay(\psi_D=0,\psi_M>0)} = \beta M_D\frac{\beta(2M_M+M_R)}{4},$$

which leads to (31).

Using (28) and (29) in (7) we have

$$M_{Pay(\psi_D=0,\psi_M>0)} = \beta M_M\frac{\beta(2M_M+M_R)}{4} - \left[\frac{\beta(2M_M+M_R)}{4}\right]^2\left[\frac{2M_M-M_R}{2M_M+M_R}\right]$$

which leads to (32). ■

With the manufacturer or distributor participating in retail advertising we observe that retail advertising does not only depend on the retail margin as in the situation where neither the manufacturer nor the distributor participates in retail advertising, but also on the participating player's margin. Further, in these cases the players' payoffs do not have only the retailer's margin as a common determinant. Instead, the participating player's margin is central to every player's payoff. The implication of these is that the participating player's subsidy is very influential on all the other players.

Proposition 5.4 (Equilibrium Characterising the Provision of Subsidy by both the Manufacturer and Distributor)

Suppose that both the distributor and the manufacturer participate in retail advertising then the distributor, manufacturer and retailer's strategies are given by

$$\psi_D = \frac{2M_D-M_R}{2(M_D+M_R)}, \tag{33}$$

$$\psi_M = \frac{2M_M-M_R}{2(M_M+M_R)} \tag{34}$$

and

$$\alpha_{(\psi_D,\psi_M>0)} = \left[\frac{\beta M_RL}{K}\right]^2 \tag{35}$$

respectively, and the resulting payoffs are

$$R_{Pay(\psi_D,\psi_M>0)} = \frac{\beta^2M_R^2L}{2K}, \tag{36}$$

$$D_{Pay(\psi_D,\psi_M>0)} = \frac{\beta^2 M_R L[2M_D K - M_R(2M_D - M_R)(M_M + M_R)]}{2K^2}, \quad (37)$$

$$M_{Pay(\psi_D,\psi_M>0)} = \frac{\beta^2 M_R L[2M_M K - M_R(2M_M - M_R)(M_D + M_R)]}{2K^2}. \quad (38)$$

where

$L = (M_D + M_R)(M_M + M_R)$,

$K = 2(M_D + M_R)(M_M + M_R) - (2M_D - M_R)(M_M + M_R) - (2M_M - M_R)(M_D + M_R)$.

Proof: Since $\psi_D \neq 0$ and $\psi_M \neq 0$ we have that

$$\psi_D = \frac{2M_D - M_R}{2(M_D + M_R)},$$

$$\psi_M = \frac{2M_M - M_R}{2(M_M + M_R)}.$$

Using (33) and (34) in (8) we have

$$\alpha_{(\psi_D,\psi_M>0)} = \frac{\beta^2 M_R^2}{4\left(1 - \frac{2M_D - M_R}{2(M_D + M_R)} - \frac{2M_M - M_R}{2(M_M + M_R)}\right)^2}$$

$$= \frac{\beta^2 M_R^2}{4}\left[\frac{2(M_D + M_R)(M_M + M_R)}{K}\right]^2,$$

where

$L = (M_D + M_R)(M_M + M_R)$,

$K = (M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)$.

This leads to (35).

Using (33), (34) and (35) in (5) we have

$$R_{Pay(\psi_D,\psi_M>0)} = \frac{\beta^2 M_R^2 (M_D + M_R)(M_M + M_R)}{(M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)}$$
$$- \left[\frac{\beta M_R (M_D + M_R)(M_M + M_R)}{(M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)}\right]^2 \times [1 - \psi_D - \psi_M]$$
$$= \frac{\beta^2 M_R^2 L}{K} - \left[\frac{\beta M_R L}{K}\right]^2 \left[\frac{K}{2L}\right]$$

which leads to (36).

Using (33) and (35) in (6) we have

$$D_{Pay(\psi_D,\psi_M>0)} = \frac{\beta^2 M_D M_R (M_D + M_R)(M_M + M_R)}{(M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)}$$
$$- \left[\frac{\beta M_R (M_D + M_R)(M_M + M_R)}{(M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)}\right]^2 \times \left[\frac{2M_D - M_R}{2(M_D + M_R)}\right]$$
$$= \frac{\beta^2 M_R (M_D + M_R)[2M_D (M_M + M_R)K - M_R (M_M + M_R)^2 (2M_D - M_R)]}{2K^2}$$

which leads to (37).

Using (34) and (35) in (7) we have;

$$M_{Pay(\psi_D,\psi_M>0)} = \frac{\beta^2 M_M M_R (M_D + M_R)(M_M + M_R)}{(M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)}$$
$$- \left[\frac{\beta M_R (M_D + M_R)(M_M + M_R)}{(M_M + M_R)[2(M_D + M_R) - (2M_D - M_R)] - (2M_M - M_R)(M_D + M_R)}\right]^2 \times \left[\frac{2M_M - M_R}{2(M_M + M_R)}\right]$$
$$= \frac{\beta^2 M_R (M_M + M_R)(M_D + M_R)[2M_M K - \beta^2 M_R (M_D + M_R)(2M_M - M_R)]}{2K^2}$$

which leads to (38). ■

Considering (35) we observe that with the participation of both the manufacturer and the distributor in retail advertising, the advertising effort depends on all the players' margins. This is a departure from a situation where there is no subsidy (in which only the retail margin is central to the advertising effort and payoffs); or a situation where only one player provides subsidy for retail advertising (in which only the participating player's margin is central to the advertising effort and payoffs).

Results and Discussion

In this section we illustrate the effect of retail advertising and subsidy strategies on the payoffs. To effectively achieve these it is necessary that the parameter values are in consonance with the model requirements. We recall that the advertising effectiveness $\beta \in [0,1]$ measures the response to advertising. Thus we let $\beta = 0.3$. Being the Stackelberg leader, the manufacturer enjoys first mover's advantage. Thus his margin is considered to be the largest. This is followed by that of the distributor who is the first follower. The retailer is the last follower, and so his margin is considered to be the least. Thus we have that $M_M > M_D > M_R$. Hence we let $M_M = 8$, $M_D = 4$, $M_R = 2$.

From Fig. 1 we observe that as the distributor's participation (subsidy) increases, both the manufacturer and the retailer's payoffs increase, and become unbounded as the participation becomes total. On the other hand the distributor's payoff reduces with his participation. We observe a similar scenario in Fig. 2 where the distributor and the retailer's payoffs increase with the manufacturer's participation, while the manufacturer's payoff reduces with his participation. The exception is that at the manufacturer's optimal participation rate, his payoff is larger than both followers' payoffs, while the distributor's payoff is larger than only the retailer's payoff.

In Fig. 1, it is clear that $M_{pay} > D_{pay}, R_{pay}\ \forall\ \psi_D$ and $D_{pay} > R_{pay}\ \forall\ \psi_D \in [0,\ 0.4000)$ (based on our choice of parameter values). Also in Fig. 2 $M_{pay} > D_{pay}\ \forall\ \psi_M \in [0,\ 0.6875)$ and $M_{pay} > R_{pay}\ \forall\ \psi_M \in [0,\ 0.7368)$ (also based on our choice of parameter values). M_{pay} being larger than D_{pay} and R_{pay} is quite understandable since the manufacturer enjoys the first mover's advantage as the Stackelberg leader. Similarly, D_{pay} being larger than R_{pay} is the advantage of being the first follower.

Thus, the distributor should not provide subsidy above 0.4000 if his payoff must be larger than that of the retailer. Similarly, the manufacturer must not provide subsidy above 0.6875 if his payoff must be above that of the distributor, neither should it be above 0.7368 if his payoff must be above that of the retailer. In essence, both the manufacturer and the distributor should stick to their optimal subsidy strategies if they would not want to be short-changed.

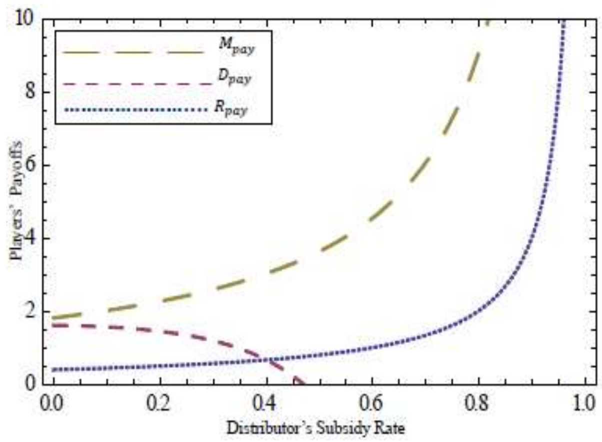

Fig. 1: The effect of the distributor's subsidy rate on the payoffs

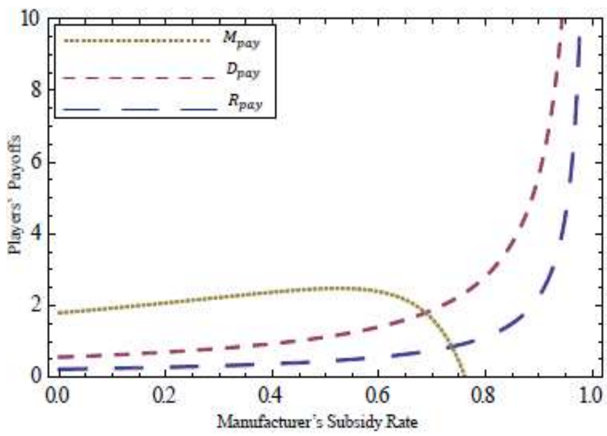

Fig. 2: The effect of the manufacturer's subsidy rate on the payoffs

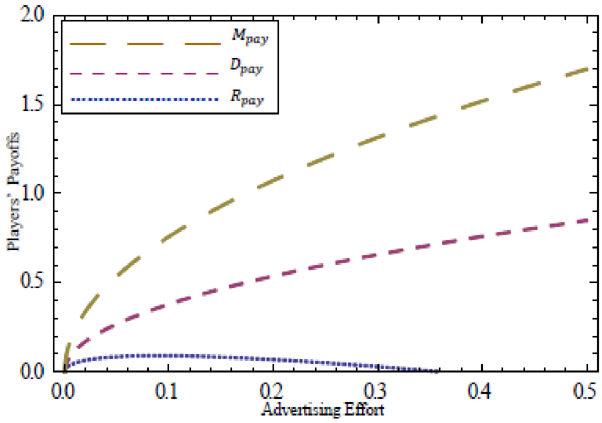

Fig. 3: The effect of the advertising effort on the payoffs in the absence of subsidy

From Fig. 3, we observe that both the manufacturer and the distributor enjoy a field day with increasing retail advertising for which they do not participate. On the other hand the retailer experiences a diminishing return only after a marginal increase which is not comparable to neither that of the manufacturer nor the distributor. We observe a similar trend in Figs. 4, 5 and 6; except that with any player's direct or indirect involvement in advertising (say through subsidy), he experiences a diminishing marginal return after a certain level of increase in his payoff as the advertising effort increases, while the none-participating player experiences a continuous payoff increase.

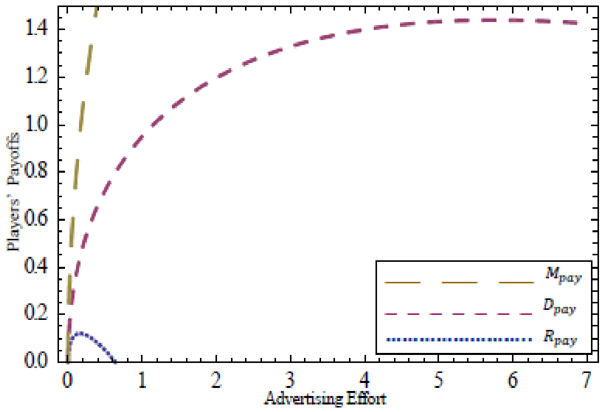

Fig. 4: The effect of the advertising effort on the payoffs when only the distributor subsidises retail advertising

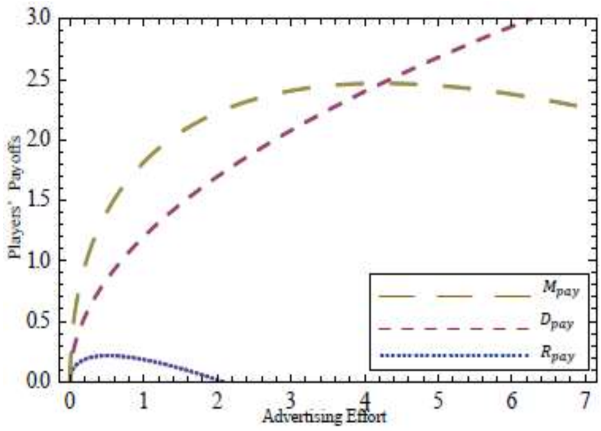

Fig. 5: The effect of the advertising effort on the payoffs when only the manufacturer subsidises retail advertising

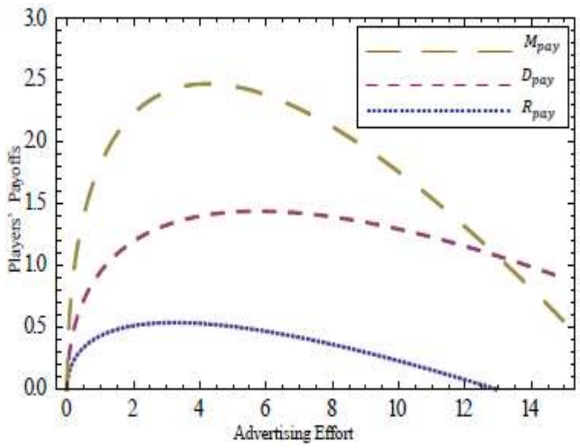

Fig. 6: The effect of the advertising effort on the payoffs when both the manufacturer and the distributor subsidises the retail advertising effort

Clearly, from Fig. 3 to 6, we observe that the retailer's payoff attains its optimum with a much lower effort while the manufacturer and the distributor need a much larger retail advertising effort to get to their optima. Thus, being that the retail advertising effort is the retailer's decision variable and within his control, he should adopt only his optimal effort that corresponds with the channel adopted structure to avoid being short-changed. However, if the manufacturer and the distributor wish that the retailer raises the advertising level for them to attain their optimal payoffs, then there has to be an agreement between the players to ensure that the extra payoff from the manufacturer and/or distributor is equitably shared in such a way that ensures that the retailer is not short-changed.

Conclusion

In this work we set out to develop for the first time a static model on cooperative advertising in a manufacturer-distributor-retailer supply chain in which only the retailer engages in advertising, while the manufacturer and distributor participate in retail advertising by providing subsidy to the retailer to aid advertising. The players engage in Stackelberg game. The work considered a four-game scenario: a channel structure without subsidy from neither of the manufacturer nor the distributor; two single-player subsidised channel structures with only the manufacturer or the distributor providing subsidy; and a two-player subsidised channel structure with subsidy from both the manufacturer and the distributor. Based on these channel structures we obtained the optimal advertising efforts, the subsidy (participation) rates, and the payoffs for all the players. We considered the effect of the players' strategies on the payoffs, and observed that every player's payoff increases with commitment to his strategy, with diminishing return setting in after a certain level. It is therefore most appropriate for each player to stick to his optimal strategy if he must not be short-changed.

This work has some limitations and possible extensions. We considered cooperative advertising game models with the manufacturer as the Stackelberg leader and the distributor and retailer as followers. An innovation can consider a situation where either the retailer or the distributor is the Stackelberg leader. Both the manufacturer and distributor participated in retail advertising. A consideration where the manufacturer can participate in retail advertising through the distributor or completely bypass the distributor to support the retailer can be very insightful.

Conflict of Interest

The author declares that there is no conflict of interest reported on this work.

References

Berger PD 1972. Vertical cooperative advertising ventures. *J. Marketing Res.* 9(3): 309-312.

Chutani A & Sethi SP 2012. Cooperative advertising in a dynamic retail market oligopoly. *Dynamic Games and Applications*, 2(4): 347-375.

Dant RP & Berger PD 1996. Modeling cooperative advertising decisions in franchising. *The J. Operational Res. Soc.*, 47(9): 1120-1136.

Ezimadu PE 2017. Modeling cooperative advertising in a static manufacturer-retailers market duopoly. *Nig. Annals of Natural Sci.*, 16(1): 95-99.

Ezimadu PE 2019. A Game-theoretic cooperative advertising model: The feasibility of the distributor's involvement in a manufacturer-distributor-retailer channel. *FUW Trends in Sci. and Techn. J.*, 4(2): 416-421.

He X, Prasad A & Sethi SP 2009. Cooperative advertising and pricing in a dynamic stochastic supply Chain: Feedback Stackelberg Strategies. *Production and Operations Management*, 18(1): 78-94.

He Y, Liu Z & Usman K 2014. Coordination of Cooperative advertising in a two-period fashion and textiles supply chain. *Mathematical Problems in Engineering*, Article ID 356726, 10 pages.

Huang Z & Li SX 2001. Co-op advertising models in a manufacturer-retailer supply chain: A game theory approach. *Euro. J. Operational Res.*, 135(3): 527-544.

Huang Z, Li SX & Mahajan V 2002. An analysis of manufacturer-retailer supply chain coordination in cooperative advertising. *Decision Science*, 33(3): 469-494.

Li SX, Zhu J & Chau PYK 2002. Cooperative advertising, game theory and manufacturer-retailer supply chains. *Omega*, 30(5): 347-357.

Xie J & Neyret A 2009. Co-op advertising and pricing models in manufacturer-retailer supply chains. *Computers and Ind. Engr.*, 56(4): 1375-1385.

Xie J & Wei J 2009. Coordination advertising and pricing in a manufacturer-retailer channel. *Euro. J. Operational Res.*, 197(2): 785-791.

Yue J, Austin J, Wang M & Huang Z 2006. Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *Euro. J. Operational Res.*, 168: 65-85.